Credit Ratings
Estimating a Credit Spread

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In this white paper we will calculate the credit spread for the debt of a hypothetical company and then compare that calculated credit spread to the actual market credit spread. We will define the market credit spread to be the difference between the yield on a corporate bond and the yield on a US Treasury note or bond of equivalent duration.

Spreads on corporate bonds tend to be many times wider than what would be implied by expected default losses alone. While credit spreads are often generally understood as the compensation for credit risk, it has been difficult to explain the precise relationship between spreads and such risk. The wide gap between spreads and expected default losses is what we call the credit spread puzzle. Market credit spreads are comprised of expected default losses, taxes, a risk premium, and a liquidity premium. [4]

We will use the following table of average default rates, recovery rates, and credit spreads from a previous white paper... [1]

Table 1: Average Default Rates, Recovery Rates, and Credit Spreads

Credit	Default Rate		Credit Spread		Recovery
Rating	Cumul	Annual	Mean	Std Dev	Rate
AAA	0.80%	0.04%	0.78%	0.52%	69.58%
AA	2.26%	0.11%	0.97%	0.66%	43.18%
A	5.53%	0.28%	1.29%	0.84%	44.17%
BBB	9.65%	0.51%	1.97%	1.02%	43.52%
BB	28.71%	1.69%	3.57%	1.77%	41.59%
B	48.71%	3.34%	5.40%	2.44%	38.36%
CCC/C	52.53%	3.73%	11.23%	5.24%	38.86%

Our task is to map a hypothetical company's credit rating to the table above so as to derive a credit spread for a hypothetical company's debt. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are given the following model parameters for our hypothetical company... [2]

Table 2: Model Parameters

Description	Symbol	Value
Asset value at time zero	A_0	10,000,000
Asset return - mean	μ	0.0953
Asset return - volatility	σ_A	0.3500
Annual dividend yield	ϕ	0.0513
Risk-free rate	α	0.0368
Lender rate markup	Δ	0.0144
Debt term in years	T	5.0000

Total return is comprised of asset price appreciation (capital gains) plus dividend income (dividend yield). Using the table above, the equation for expected capital gains is $\mu - \phi$.

The table below presents the leverage ratios for our hypothetical company for a 5 year debt term... [2]

Table 3: Leverage Ratios

Credit Rating	Default Rate	Default Point	Leverage Ratio
AAA	0.04%	-2.34	10.73%
AA	0.11%	-2.07	14.07%
A	0.28%	-1.80	18.31%
BBB	0.51%	-1.62	22.00%
BB	1.69%	-1.18	34.14%
B	3.34%	-0.88	45.86%

The debt leverage ratio in column four is consistent with the default point in column three, which in turn is consistent with the annual default probability in column two.

Enterprise Value

We will define the variable T to be debt term in years, the variable A_T to be asset value (i.e. enterprise value) at time T, the variable α to be the risk-free rate, the variable ϕ to be the dividend yield, the variable σ_A to be asset return volatility, and the variable z to be a normally-distributed random variable with mean zero and variance one. The equation for random asset value at time T under the risk-neutral probability Measure Q is...

$$A_T = A_0 \,\mathrm{Exp}\left\{\left(\alpha - \phi - \frac{1}{2}\,\sigma_A^2\right)T + \sigma_A \sqrt{T}\, z\right\} \text{ ...where... } z \sim N\Big[0, 1\Big] \tag{1}$$

We will define the variable n to be asset return mean under the risk-neutral probability Measure Q and the variable v to be asset return variance. The equations for return mean and variance over the time interval $[0, T]$ are...

$$n = \left(\alpha - \phi - \frac{1}{2}\,\sigma^2\right)T \text{ ...and... } v = \sigma_A^2 T \tag{2}$$

Using the definitions in Equation (2) above, we can rewrite random asset value Equation (1) above as...

$$A_T = A_0 \,\mathrm{Exp}\Big\{\theta\Big\} \text{ ...where... } \theta \sim N\Big[n, v\Big] \tag{3}$$

Using Equations (2) and (3) above, the equation for expected asset value at time T under the risk-neutral probability Measure Q is...

$$\mathbb{E}^Q\Big[A_T\Big] = \int\limits_{-\infty}^{\infty} \sqrt{\frac{1}{2\,\pi\, v}}\,\mathrm{Exp}\left\{\frac{1}{2\,v}\,(\theta - n)^2\right\} A_0 \,\mathrm{Exp}\Big\{\theta\Big\}\,\delta\theta = A_0 \,\mathrm{Exp}\left\{\Big(\alpha - \phi\Big)T\right\} \tag{4}$$

Debt Value

We will assume that the asset price path is log-linear between the known start point A_0 and the random end point A_T. We will define the variable t to be a time variable that has a value between zero and T. Using Equation (3) above, the equation for random asset value at time t is...

$$A_t = A_0 \,\mathrm{Exp}\left\{\frac{\theta}{T}\,t\right\} \text{ ...where... } 0 \le t \le T \tag{5}$$

We will define the variable D_t to be debt value at time t and the variable Γ to be the debt leverage ratio. The equation for the debt leverage ratio is...

$$\Gamma = \frac{D_0}{A_0} \text{ ...such that... } D_0 = \Gamma A_0 \tag{6}$$

We will define the variable Δ to be lender's risk-free rate markup such that the equation for the interest rate on the debt is...

$$\text{Debt interest rate} = \alpha + \Delta \tag{7}$$

We will define the variable β to be the default point factor. We will assume that the borrower will service the debt up until the point where random asset value is below the default poiint. Using Equations (5) and (6) above, the borrower will default at time t when the following equality holds...

$$\text{if... } A_0 \, \text{Exp}\left\{\frac{\theta}{T}\,t\right\} = \beta\, D_0 = \beta\,\Gamma A_0 \text{ ...then... } t = T\ln(\beta\,\Gamma)\,\theta^{-1} \tag{8}$$

When the borrower defaults on debt service payments then accrued interest is added to the debt. Using Equations (6), (7) and (8) above, the equation for debt value at time T given that there was a default is...

$$D_T = D_0 \, \text{Exp}\left\{(\alpha + \Delta)\,(T - t)\right\} \tag{9}$$

We will define the variable LGD to be the nominal loss given default and the variable ω to be the post-default recovery rate. The equation for loss given default is...

$$LGD = (1 - \omega)\,\text{Debt principal} + \text{Accrued interest} \tag{10}$$

Using Equations (6) and (9) above, we can rewrite Equation (10) above as...

$$LGD = (1 - \omega)\,D_0 + (D_T - D_0) \tag{11}$$

Using Appendix Equation (23) below, the solution to Equation (11) above is...

$$LGD = \Gamma A_0 \left(\text{Exp}\left\{(\alpha + \Delta)\,(T - t)\right\} - \omega\right) \text{ ...where... } [t] \text{ is a normally-distributed random variable} \tag{12}$$

Using Appendix Equation (24) below, we can rewrite Equation (12) above as...

$$LGD = \Gamma A_0 \left(\text{Exp}\left\{(\alpha + \Delta)\,T\left[1 - \ln(\beta\,\Gamma)\,\theta^{-1}\right]\right\} - \omega\right) \text{ ...where... } \theta \sim N\left[n, v\right] \tag{13}$$

Credit Spread

The equation for the value of the credit default swap is...

$$CDS = \int\limits_{-\infty}^{a} \sqrt{\frac{1}{2\,\pi\,v}}\,\text{Exp}\left\{\frac{1}{2\,v}\,(\theta - n)^2\right\} LGD\,\text{Exp}\left\{-\,\alpha\,T\right\}\delta\theta \tag{14}$$

Using Equation (13) above, we can rewrite Equation (14) above as...

$$CDS = \int\limits_{-\infty}^{a} \sqrt{\frac{1}{2\,\pi\,v}}\,\text{Exp}\left\{\frac{1}{2\,v}\,(\theta - n)^2\right\} \Gamma A_0 \left(\text{Exp}\left\{(\alpha + \Delta)\,T\left[1 - \ln(\beta\,\Gamma)\,\theta^{-1}\right]\right\} - \omega\right) \text{Exp}\left\{-\,\alpha\,T\right\}\delta\theta \tag{15}$$

Note that there is no closed-form solution to the integral in Equation (15) above. The solution to that integral must be calculated using numerical methods such as Numerical Integration (preferred) or Monte Carlo.

The Answers To Our Hypothetical Problem

Using the equations above, our credit spread table is presented below...

Table 4: Credit Spreads

Credit	Credit Spread		
Rating	Actual	CDS	Ratio
AAA	0.78%	0.04%	19.68
AA	0.97%	0.18%	5.25
A	1.29%	0.41%	3.16
BBB	1.97%	0.69%	2.85
BB	3.57%	2.07%	1.72
B	5.40%	3.99%	1.35

Note that in the table above the first column is the letter credit grade, the second column is the market credit spread, the third column is the credit spread that we calculate using the CDS equations above, and the fourth column is the ratio of the market credit spread to the calculated CDS credit spread.

We will make the following observations...

1. The ratio of the market credit spread to the CDS calculated credit spread decreases as the credit rating decreases from AAA to B.
2. The decrease in this ratio is most likely due to income tax and capital funding differences between US Treasuries and corporate bonds plus a liquidity premium.
3. The differences noted in item 2 above most likely have a large fixed component plus a smaller variable component such that the credit spread ratio decreases as the credit rating decreases.
4. As the debt term increases the credit spread ratio decreases.

Example Calculation - B Credit Rating

Using Equation (2) above and the data in Table 2 above, the equations for the risk-neutral return mean (m) and variance (v) are...

$$n = \left(0.0368 - 0.0513 - \frac{1}{2} \times 0.3500^2\right) \times 5.00 = -0.3786 \text{ ...and... } v = 0.3500^2 \times 5.00 = 0.6125 \tag{16}$$

We will define the variable a to be the default point, which is the asset return where asset value equals the default threshold. The equation for the default point is...

$$a = \text{Default point from credit rating Table 3 above} = -0.88 \tag{17}$$

The equation for the debt leverage ratio is...

$$\Gamma = \text{Debt leverage ratio from Table 1 above} = 0.4586 \tag{18}$$

The equation for the recovery rate is...

$$\omega = \text{Recovery rate from Table 1 above} = 0.3886 \tag{19}$$

Using Table 2 above, we will make the following variable definitions...

$$\alpha = 0.0368 \text{ ...and... } \Delta = 0.0144 \text{ ...and... } T = 5.00 \tag{20}$$

Using Equation (15) above and the parameter definitions in Equations (16) to (20) above, the equation for the value of the credit default swap at time zero is...

$$\begin{aligned} CDS = & \int\limits_{-\infty}^{a} \sqrt{\frac{1}{2 \times \pi \times 0.6125}} \text{Exp}\left\{\frac{1}{2 \times 0.6125}(\theta + 0.3786)^2\right\} \times 0.4586 \times 10,000,000 \\ & \times \left(\text{Exp}\left\{(0.0368 + 0.144) \times 5.00 \times \left[1 - \ln(0.9000 \times 0.4586) \times \theta^{-1}\right]\right\} - 0.3886\right) \\ & \times \text{Exp}\left\{-0.0368 \times 5.00\right\} \delta\theta \end{aligned} \tag{21}$$

After solving Equation (21) above analytically, the value of the credit default swap for B rated debt is...

$$CDS = \$690,128 \tag{22}$$

The credit spread is the difference between the debt yield before and after the guarantee (CDS). The credit spread is... [3]

Description	Rate	Notes
Debt yield before guarantee	5.25%	Exp {IRR in continuous-time} − 1
Debt yield after guarantee	9.24%	Exp {IRR in continuous-time} − 1
Credit spread	3.99%	Discrete-time spread

Appendix

A. Using Equations (6) and (9) above, the solution to Equation (11) above is...

$$\begin{aligned} LGD &= (1-\omega)\,D_0 + (D_T - D_0) \\ &= (1-\omega)\,D_0 + \left(D_0 \operatorname{Exp}\left\{(\alpha+\Delta)\,(T-t)\right\} - D_0\right) \\ &= D_0 - \omega\,D_0 + D_0 \operatorname{Exp}\left\{(\alpha+\Delta)\,(T-t)\right\} - D_0 \\ &= D_0\left(\operatorname{Exp}\left\{(\alpha+\Delta)\,(T-t)\right\} - \omega\right) \\ &= \Gamma A_0\left(\operatorname{Exp}\left\{(\alpha+\Delta)\,(T-t)\right\} - \omega\right) \end{aligned} \tag{23}$$

B. Using Equation (8) above, we can rewrite Equation (12) as...

$$\begin{aligned} LGD &= \Gamma A_0\left(\operatorname{Exp}\left\{(\alpha+\Delta)\,(T-t)\right\} - \omega\right) \\ &= \Gamma A_0\left(\operatorname{Exp}\left\{(\alpha+\Delta)\left[T - T\,\ln(\beta\,\Gamma)\,\theta^{-1}\right]\right\} - \omega\right) \\ &= \Gamma A_0\left(\operatorname{Exp}\left\{(\alpha+\Delta)\,T\left[1 - \ln(\beta\,\Gamma)\,\theta^{-1}\right]\right\} - \omega\right) \end{aligned} \tag{24}$$

References

[1] Gary Schurman, *Credit Ratings - Default Rates, Recovery Rates and Credit Spreads (Part I),* June, 2023.

[2] Gary Schurman, *Credit Ratings - The Debt Leverage Ratio (Part II),* June, 2023.

[3] Gary Schurman, *Modeling Debt - Solving for Debt IRR,* March, 2023.

[4] BIS Quarterly Review, *The Credit Spread Puzzle,* December, 2003.